

## TEMPERATURE PROFILE IN THE STATIONARY AND NONSTATIONARY REGIMES OF HEATING AND MASS REMOVAL OF LOW-THERMAL-CONDUCTIVITY MATERIALS

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*Based on an analytical solution of the linear equation of heat conduction with mass removal and without it and on the regularity of reaching the stationary heating regime which is established, when the thicknesses of the heated and removed material layers are equal, the authors have proved the S-shaped temperature profile found experimentally near the destroyed surface. It has been noted that the use of the readings of a temperature sensor located in the S-shaped zone of the temperature profile as the boundary conditions can bring about substantial errors in determining thermophysical characteristics using inverse heat-conduction problems.*

The main methods of determination of the thermophysical characteristics of materials, particularly at high temperatures, are based on solution of inverse heat-conduction problems (see, for example, [1, 2]). The thermophysical properties are reconstructed, as a rule, from the results of experimental measurement of the temperature profile at several points of a sample. Since the results obtained are largely dependent on the site where temperature sensors (thermocouples) are installed, special methods of experimental design, which allow reliable results with a minimum number of sensors, have been developed [3, 4].

In solving the inverse coefficient heat-conduction problem (ICHCP), one usually considers a temperature field obtained at three or four points of the material's sample. The readings of the first and last thermocouple are used as the boundary conditions, as a rule. The closer the thermocouple to the heated surface, the higher the temperatures at which one is able to determine the thermophysical properties. The temperature near the surface of thermal protective materials is often higher than the working range of the thermocouples, and a thermocouple can reach the surface destroyed in mass removal at all. Clearly, under such conditions of heating, one is unable to reconstruct the thermophysical properties at destroyed-surface temperature. At the same time, it is assumed that if the material temperature is no higher than the working range of a thermocouple and it remains within the material during the experiment, the temperature fields obtained enable one to determine the thermophysical properties with a satisfactory accuracy. In this case, when there are no physicochemical transformations in the material, to determine the effective thermophysical characteristics using ICHCPs one usually uses the heat-conduction equation in the form

$$\rho c \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right). \quad (1)$$

In experimental investigations of the temperature fields in quartz glass ceramics (QGC), we have established an S-shaped temperature profile near the destroyed surface [5]; the occurrence of this temperature profile in homogeneous materials of the quartz-glass-ceramics type was attributable only to the considerable increase in the thermal conductivity due to the radiant component of heat transfer. We carried out computational-experimental investigations of the temperature fields in samples of pure and chromic-oxide-doped quartz glass ceramics. Unlike pure quartz glass ceramics whose surface layer becomes semitransparent at temperatures higher than 2000 K, quartz glass ceramics doped with chromic oxide acquires a green color and becomes opaque, in practice, at temperatures higher than 1800 K [6].

Indeed, the numerical calculations from the equation

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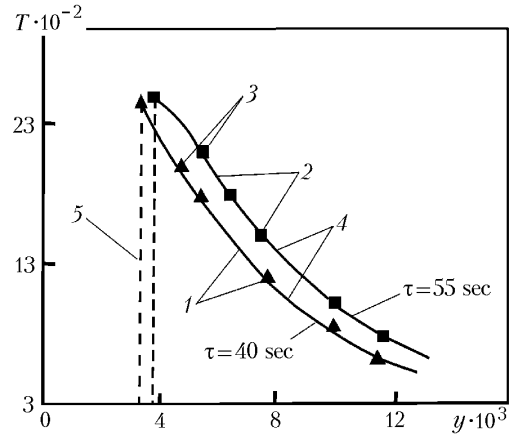
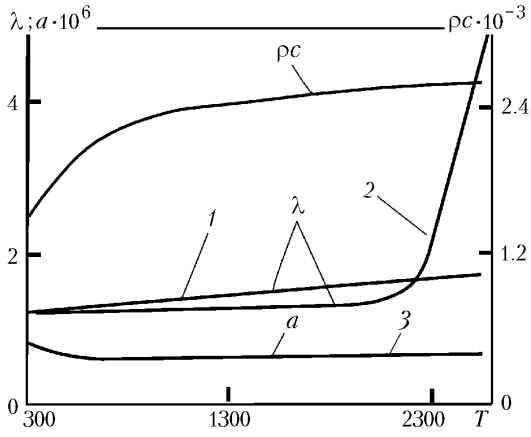


Fig. 1. Thermophysical characteristics of quartz glass ceramics: 1 and 3) doped ceramics; 2) pure ceramics.  $\lambda$ , W/(m·K);  $\rho c$ , kJ/(m<sup>3</sup>·K);  $a$ , m<sup>2</sup>/sec;  $T$ , K.

Fig. 2. Comparison of the temperature profiles in pure and doped QGCs: 1) doped QGC at the 40th second of heating ( $q_{\text{cal}} = 8350$  kW/m<sup>2</sup>,  $T_w = 2390$  K, and  $V_{\infty} = 0.1 \cdot 10^{-3}$  m/sec); 2) pure QGC at the 55th second of heating ( $q_{\text{cal}} = 7260$  kW/m<sup>2</sup>,  $T_w = 2510$  K, and  $V_{\infty} = 0.08 \cdot 10^{-3}$  m/sec); 3) experiment; 4) calculation from (2) [7]; 5) position of the heated surface.  $T$ , K;  $y$ , m.

$$\rho c \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \rho c V_{\infty} \frac{\partial T}{\partial y} \quad (2)$$

with the corresponding boundary conditions [7] and the thermophysical characteristics given in Fig. 1 are in good agreement with experimental data and confirm the occurrence of an S-shaped profile in the pure quartz glass ceramics (Fig. 2, curve 2), whose thermal conductivity nearly trebles due to the radiant component of heat transfer at temperatures higher than 2000 K (Fig. 1, curve 2). In the stationary regime of heating of the quartz glass ceramics doped with chromic oxide, the temperature profile is exponential in character, as could be expected, and is well described by an equation of the form

$$\theta(y) = \frac{T(y) - T_0}{T_w - T_0} = \exp \left( - \frac{\bar{V}_{\infty}}{a} y \right), \quad (3)$$

which follows from (2) for constant thermophysical properties and  $\tau \rightarrow \infty$ .

However, the S-shaped temperature profile near the destroyed surface was also established in the nonstationary regime of heating of samples of the opaque doped quartz glass ceramics whose thermal diffusivity is virtually independent of temperature (Fig. 1, curve 3). Accordingly, the calculations for this heating zone from (1) and (2) were not confirmed by experimental data. Furthermore, it was shown that the reconstruction of thermal conductivity when the readings of a thermocouple appearing in the S-shaped zone of the temperature profile (this zone is the most pronounced at high mass-removal velocities) are used as boundary conditions can lead to a sharp (two-times) reduction in the thermal conductivity (Fig. 3, curve 2).

The solution of the linear heat-conduction equation for a half-space

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

has been obtained in [8]; with the boundary conditions of the first kind, for example,  $T_w = \text{const}$ , it has the form

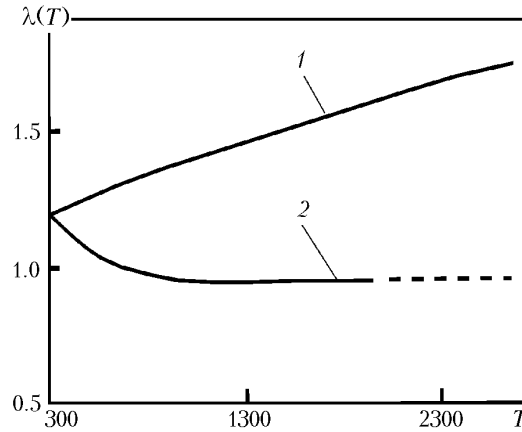


Fig. 3. Thermal conductivity of a doped QGC reconstructed with the use of the readings of internal (first and fourth) thermocouples as the boundary conditions: for  $V_\infty = 0.1 \cdot 10^{-3}$  (1) and  $0.02 \cdot 10^{-3}$  m/sec (2).

$$\theta^* = \frac{T^* - T_0}{T_w - T_0} = \operatorname{erfc} \left( \frac{y}{2\sqrt{a\tau}} \right). \quad (5)$$

The heated-layer depth bounded by the isotherm with a dimensionless temperature  $\theta^*$  obeys the relation

$$y \approx K\sqrt{a\tau}, \quad (6)$$

where the coefficient  $K$  characterizes the velocity of movement of different isotherms and is dependent only on  $\theta^*$  and the law of variation in  $T_w$  [9]. However, it has been established that if the position of the isotherm with a dimensionless temperature  $\theta^*$  is considered relative to the initial material surface, a relation analogous to (6) is satisfactorily accurate for low destruction rates ( $V_\infty < 0.05 \cdot 10^{-3}$  m/sec) and in the presence of the internal gasification front (decomposition of the binder). When  $\theta^* < 0.2$ , the coefficient  $K$  is calculated from (5); when  $\theta^* \geq 0.2$ , it is calculated from the formula

$$K = -\frac{1}{K_{T_d}} \theta^* + \frac{K_{T_d}^2}{1 - K_{T_d}}. \quad (7)$$

Expression (7), where a unique value of  $K_{T_d}$  is possible, becomes, for  $\theta^* = 1$ , the third-degree equation

$$2K_{T_d}^3 = K_{T_d}^2 + K_{T_d} - 1 = 0,$$

whose solution yields a numerical value of the thermal-destruction constant of  $K_{T_d} \approx 0.74$ .

Figure 4 gives, as an example, results of testing asbestos-textolite samples in air and nitrogen flows and in radiative heating; these results have been obtained for virtually the same values of the calorimetric heat flux and the temperature on the surface and clearly confirm this conclusion. Despite the fact that the velocities of mass removal differed three times due to the different mechanism of destruction under these conditions of heating, the distance from the initial surface of the samples to the lower boundary of the carbonized layer was virtually the same in the nonstationary regime (Fig. 4b). We can assume that before the establishment of the stationary regime of heating, when the velocity of the isotherm in question becomes equal to the removal velocity, the mechanism of destruction of the surface of the material influences its temperature field only slightly and the heating depth is determined by the initial value of the heat flux.

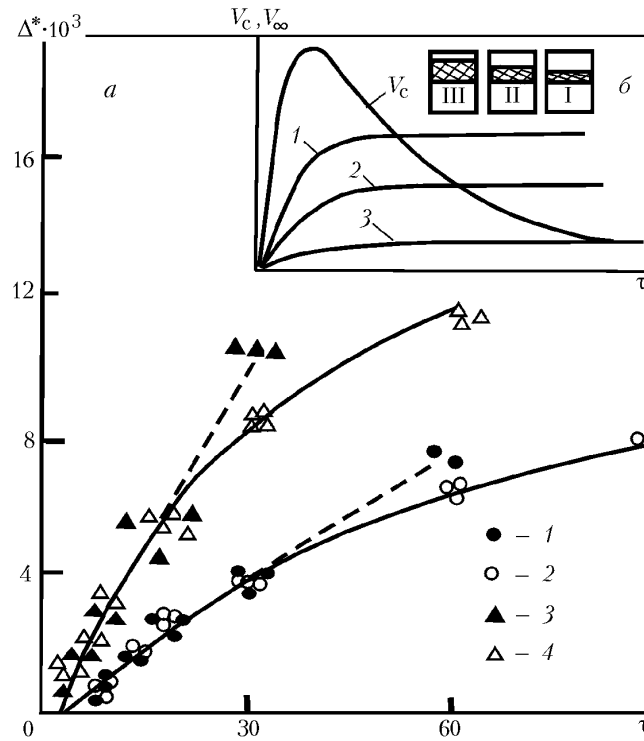


Fig. 4. Total thickness of the carbonized and removed layers vs. heating time (a) and the diagram of variation in the velocities of movement of the surface and the carbonization front in asbestos-textolite samples (b): a) 1 and 3) convective heating in an air flow; 2 and 4) radiative heating; 1 and 2) heat flux  $1600 \text{ kW/m}^2$ ; 3 and 4)  $10,000 \text{ kW/m}^2$ ; b) 1, 2, and 3) velocity of movement of the material surface in heating in air and nitrogen flows and in radiative heating; I-III, carbonized (dashed) and removed (light) layers in the asbestos-textolite samples after testing in air and nitrogen flows and in radiative heating; dashed curves, stationary regime of carbonization.  $T_{wI} = T_{wII} = T_{wIII}$ .  $\Delta^*$ , m;  $\tau$ , sec.

In [7], it has been shown that the stationary rate of destruction of the surface is attained not at  $\tau \rightarrow \infty$  but at the instant of time  $\tau_v$ , when a material layer whose thickness can be evaluated by the formula

$$S(\tau_v) = \frac{d_0}{K_{T_d}} \approx 1.83d_0, \quad (8)$$

where  $d_0$  is the parameter of nonstationary mass removal, weakly dependent on all properties of the material except its thermal conductivity, is removed from the surface [10]. It takes a time  $\tau_\delta$  (whose value can be an order of magnitude or more higher than the time  $\tau_v$  and is dependent on the dimensionless temperature of the isotherm  $\theta^* = (T^* - T_0) / (T_w - T_0)$  in question) to attain the stationary regime of heating where the velocity of movement of the isotherms of the temperature field inside the material is equal to the velocity of mass removal.

As a result of the computational-experimental investigations (generated in [7]) of the heating and mass removal of thermal protective materials, it has been shown that the process characterized by the thermal-destruction constant of the material  $K_{T_d}$  is most probably responsible for the quite rapid transition from the nonstationary regime of mass removal to a stationary one, as is for the S-shaped temperature profile. This constant determines the regularities of variation in the surface temperature in the temperature interval  $T_d - T_w$ , the velocity of mass removal, and the rate of heating in the nonstationary regime.

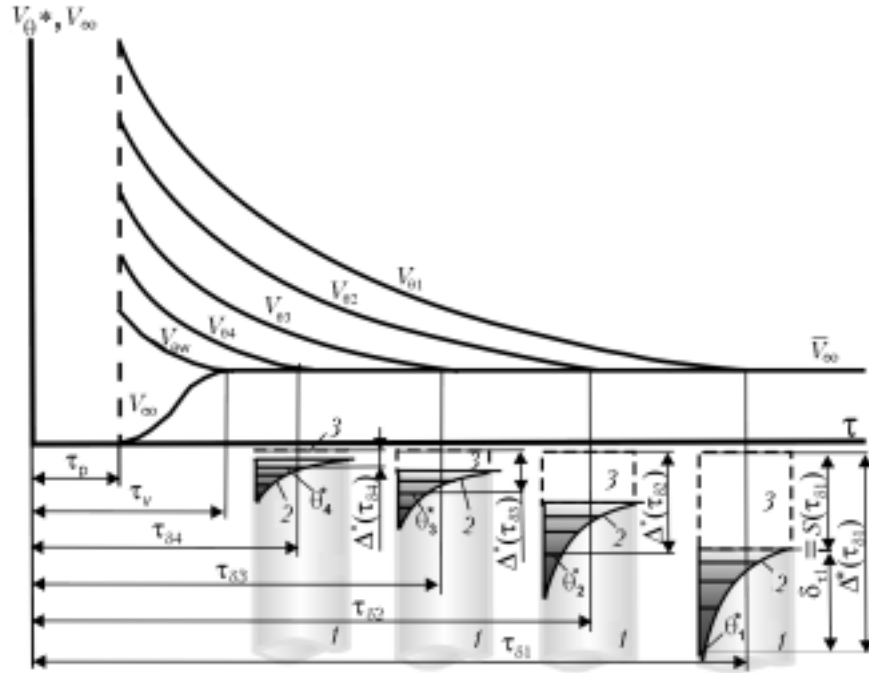


Fig. 5. Model of heating and mass removal of a thermal protective material: 1) sample; 2) temperature profile; 3) removed layer;  $\theta_1^* - \theta_4^*$ , isotherms bounding the heated layer whose thickness is equal to the removed-layer thickness.  $\theta_1^* < \theta_2^* < \theta_3^* < \theta_4^* < \theta_w^* = 1$ ;  $\delta_{\tau_1}$ , depth of the heated layer at the instant of time  $\tau_1$ .

Taking into account that, when the rates of destruction of the surface are low, Eq. (6) is in good agreement with experimental data up to the establishment of the stationary regime of heating (see, for example, Fig. 4), we can consider the solution of Eq. (4) with the corresponding boundary conditions.

In stationary destruction of the material surface ( $V_{\infty} = \text{const}$ ), we write the following initial and boundary conditions:

$$\begin{aligned}
 1) \quad & - \left( \lambda \frac{\partial T}{\partial y} \right) \Big|_{y=S(\tau)} = \rho \bar{V}_{\infty} c (\bar{T}_w - T_0) = \rho \bar{V}_{\infty} H (T_w) = \text{const}; \\
 2) \quad & T(y, \tau) \Big|_{y=S(\tau)} = \bar{T}_w; \quad 3) \quad \text{at } \tau > 0 \text{ and } y \rightarrow \infty \quad T \rightarrow T_0; \\
 4) \quad & y > S(\tau).
 \end{aligned} \tag{9}$$

Solving Eq. (4) by the method of Laplace integral transforms for initial and boundary conditions (9), we obtain a relation analogous to (3):

$$T(y, \tau) = T_0 + (\bar{T}_w - T_0) \exp(-hy + h^2 a \tau), \quad \text{where } h = \bar{V}_{\infty} / a. \tag{10}$$

Since the solution of (10) from (4) did not require that the condition  $\tau \rightarrow \infty$  be satisfied, we can use Eq. (10) at  $\tau > \tau_v$  and  $S(\tau) < y < y_S$  with allowance for the regularities obtained [7]. The position of the boundary  $y_S$  is determined by the regularity that implies that the stationary regime of heating for any isotherm of the temperature field is established at the instant of time when the depth of this isotherm ( $\delta_T$ ) is equal to the thickness of the material layer removed from the surface (Fig. 5). Thus,  $y_S \approx S(\tau) + \delta_T \approx 2S(\tau)$  is the coordinate reckoned from the initial surface and determining the lower boundary of the stationary heated layer.

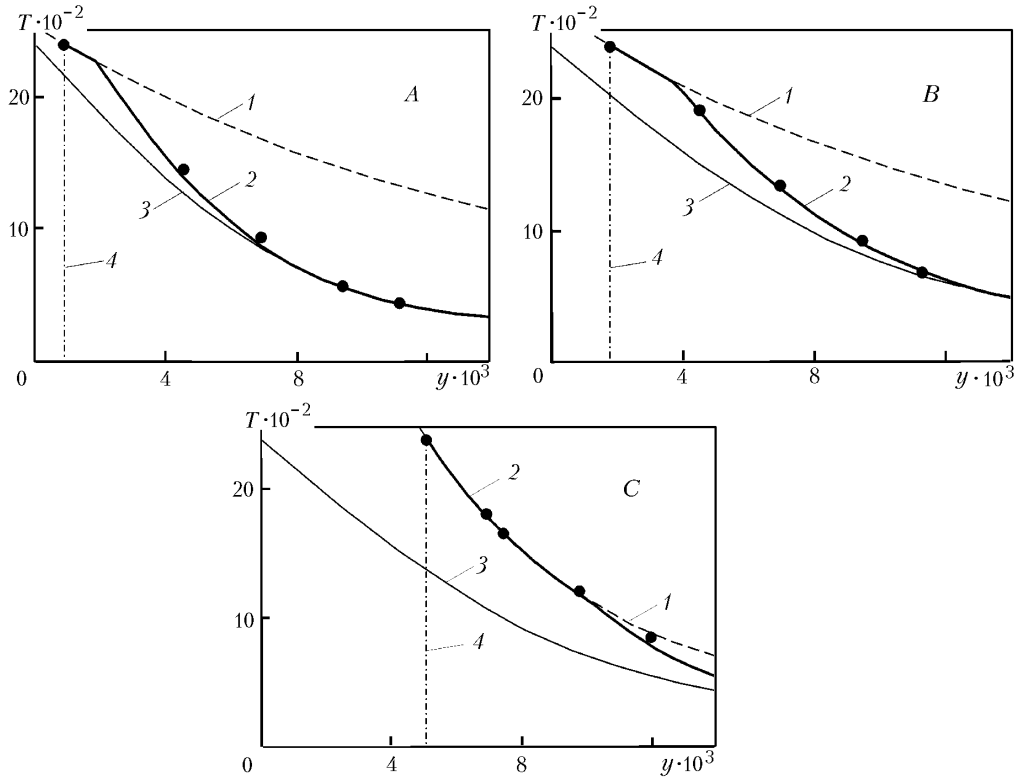


Fig. 6. Comparison of the calculated and experimental temperature profiles in a doped QGC: A and B)  $V_{\infty} = 0.05 \cdot 10^{-3}$  m/sec,  $T_w = 2400$  K, and  $a = 0.65 \cdot 10^{-6}$  m<sup>2</sup>/sec (A) at the 30th second of heating; B) at the 50th second; C)  $V_{\infty} = 0.11 \cdot 10^{-3}$  m/sec,  $T_w = 2390$  K, and  $a = 0.6 \cdot 10^{-6}$  m<sup>2</sup>/sec (at the 50th second of heating); 1) calculation from (12); 2) from (11) for  $y \geq y_S$  and from (12) for  $S(\tau) < y \leq y_S$ ; (points, experiment); 3) from (5); 4) position of the heated surface;  $T$ , K;  $y$ , m.

To solve (4) on the portion  $y > 2S(\tau)$  we should use (10) as the boundary condition at  $y = 2S(\tau)$ , taking into account that  $S(\tau) \approx V_{\infty}\tau$ :

$$1) T(y, \tau) \Big|_{y=2S(\tau)} = T_0 + (\bar{T}_w - T_0) \exp\left(-\frac{\bar{V}_{\infty}}{a} \tau\right), \quad 2) \text{ at } \tau > 0 \text{ and } y \rightarrow \infty \quad T \rightarrow T_0.$$

As a result we find

$$T(y, \tau) = T_0 + (\bar{T}_w - T_0) \exp\left(-\frac{\bar{V}_{\infty}}{a} [y - S(\tau)]\right) \left[1 - \operatorname{erf}\left(\frac{y - y_S}{2\sqrt{a\tau}}\right)\right]. \quad (11)$$

Taking into account that the removed-layer thickness is equal to  $S(\tau) = \bar{V}_{\infty}\tau - d_0$ , from (10) we write for the temperature distribution in the stationary heated layer

$$T(y, \tau) = T_0 + (\bar{T}_w - T_0) \exp\left[-\frac{\bar{V}_{\infty}}{a} (y - \bar{V}_{\infty}\tau + d_0)\right], \quad (12)$$

where  $S(\tau) < y < y_S$ .

Since the thermal diffusivity of the doped quartz glass ceramics is virtually independent of temperature (Fig. 1, curve 3), this enables us to compare the experimental data obtained under different conditions of heating and the

results of calculations from (11) and (12). The experimental procedure and that of installation of the thermocouples have been described in [5, 7].

It is noteworthy that the calculated and experimental data have been compared for the stationary regime of mass removal, i.e., for times longer than the time of establishment of the stationary rate of destruction of the surface  $\tau_v$ .

Based on the results given in Fig. 6, we can draw the following conclusions.

1. The experimental data are in good agreement with the results of calculation from (12) for  $S(\tau) < y < y_S$  and with those of calculation from (11) for  $y \geq y_S$ .

2. The S-shaped temperature profile in homogeneous materials occurs due to the rather sharp boundary line between the stationary and nonstationary regions of the temperature field. The depth of the lower boundary of the stationary heated layer is equal to the thickness of the material layer removed from the surface.

3. All the calculated and experimental results are in the domain bounded by the solutions (5) and (12).

4. In reconstructing the thermophysical properties of low-thermal-conductivity materials using ICHCPs, it is not recommended that the readings of a thermocouple appearing in the stationary region of the heated layer be used as the boundary condition if the remaining temperature sensors are at a distance longer than  $y_S$  from the initial surface.

## NOTATION

$a$ , thermal-diffusivity coefficient,  $m^2/\text{sec}$ ;  $c$ , heat capacity,  $\text{kJ}/(\text{kg}\cdot\text{K})$ ;  $d_0$ , parameter of nonstationary mass removal; it prescribes the shift of the linear removal from the origin of coordinates,  $m$ ;  $H(T_w)$ , heat content of the material at the surface temperature,  $\text{kJ}/\text{kg}$ ;  $K$ , coefficient characterizing the velocity of movement of the isotherm;  $K_{T_d}$ , thermal-destruction constant;  $q_{\text{cal}}$ , heat flux to the cold calorimeter surface,  $\text{kW}/\text{m}^2$ ;  $S(\tau)$  and  $S(\tau_v)$ , layer thickness in linear removal from the material surface and its value at the instant of establishment of the stationary removal velocity,  $m$ ;  $T$ , temperature,  $\text{K}$ ;  $T_0$ , temperature of the unheated material,  $\text{K}$ ;  $T_w$  and  $T_w$ , temperature of the heated surface and its stationary value,  $\text{K}$ ;  $T^*$ , temperature of the isotherm,  $\text{K}$ ;  $T(y)$ , running value of the temperature,  $\text{K}$ ;  $T_d$ , temperature of the beginning of destruction (melting) of the material surface,  $\text{K}$ ;  $V_c$ , velocity of the carbonization isotherm,  $m/\text{sec}$ ;  $V_\infty$  and  $V_\infty$ , linear-removal velocity and its stationary value,  $m/\text{sec}$ ;  $V_{\theta^*}$ , velocity of the isotherm,  $m/\text{sec}$ ;  $y$ , coordinate,  $m$ ;  $y_S$ , coordinate of the lower boundary of the stationary heated layer from the initial surface,  $m$ ;  $\delta_T$ , thickness of the stationary heated layer,  $m$ ;  $\theta(y)$ , running value of the dimensionless temperature;  $\theta^* = (T^* - T_0)/(T_w - T_0)$ , dimensionless temperature of the isotherm;  $\lambda$ , thermal conductivity,  $\text{W}/(\text{m}\cdot\text{K})$ ;  $\rho$ , density,  $\text{kg}/\text{m}^3$ ;  $\Delta^*$ , total thickness of the heated and removed layers to the isotherm  $T^*$ ,  $m$ ;  $\tau$ , heating time,  $\text{sec}$ ;  $\tau_d$ , time of the beginning of destruction (melting) of the surface,  $\text{sec}$ ;  $\tau_v$  and  $\tau_\delta$ , times of establishment of the stationary values of the mass-removal velocity and thickness of the heated layer,  $\text{sec}$ . Subscripts: 0, unheated material; c, carbonization; S, S-shaped temperature profile; v, velocity; w, conditions on the wall;  $\delta$ , heated layer; cal, calorimetric; d, destruction.

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